Eur. Phys. J. B **11**, 429–437 (1999)

THE EUROPEAN PHYSICAL JOURNAL B

EDP Sciences

Orbital effect of a magnetic field on two-leg Hubbard ladder

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Received 5 May 1999

Abstract. We study the effect of a magnetic field on the dimensional crossover of weakly coupled two-leg Hubbard ladders under pressure. Our model is based on the perturbative renormalization approach (PRG) with two cut-off parameters, the bandwidth E_0 and a characteristic magnetic energy ω_c. We determine the temperature-pressure phase diagram for different values of the magnetic field and discuss the relative stability of the d-wave superconducting phase (SCd) and the two dimensional Fermi liquid phase (2D) which appear at zero field. We show that the field induces a reduction in the effective dimensionality of the system and confines the electron motion within the ladder. In fact, we find that with increasing magnetic field, the isolated ladder phase gets wider at the expense of the SCd phase which disappears at a critical magnetic field H_c. The superconducting transition temperature T_c is found to decrease as the field increases up to H_c for which T_c falls to zero. Concerning the 2D phase, we show that it is destroyed for ω_c greater than the crossover temperature at which the system crosses to 2D phase at zero magnetic field.

PACS. 64.60.-i General studies of phase transitions – 64.60.Ak Renormalization-group, fractal, and percolation studies of phase transitions – 71.27.+a Strongly correlated electron systems; heavy fermions

1 Introduction

The ladder compounds which may be regarded as a crossover between one dimensional and two dimensional systems, provide a good theoretical and experimental probe to understand the properties of high T_c superconductors (HTS) especially the breakdown of the Fermi liquid theory in the normal state [1]. Therefore recent interest has been focused on these compounds which belong to spin liquid state and exhibit numerous similarities with HTS namely a spin gap and a transition from insulator to metal upon doping [2].

Despite numerous studies of ladder systems, it still remains to understand how these non-Fermi liquid quasi-one dimensional systems evolve into isotropic two dimensional behavior under coupling between ladders.

The one dimensional behavior is known to be reinforced by the orbital effect of a magnetic field as has been shown in organic conductors. The most spectacular and convincing evidence of this phenomenon is the field induced SDW in the Bechgaard salts as first discussed by Gor'kov *et al.* [3] and Héritier *et al.* [4].

In this paper we undertake an attempt to study the crossover from isolated ladders to two dimensional coupled ladder system under an applied magnetic field. We will discuss the orbital effect of the magnetic field on the phase diagram of two-leg Hubbard ladders weakly coupled by interladder hopping process. Our study is based on perturbative renormalization group theory (PRG) with two cut-off parameters which are the bandwidth E_0 and a characteristic magnetic energy ω_c.

The outline of this paper is as follows. In Section 2 we give a description of our model. In Section 3 we derive the renormalization group equations for the interladder processes. In Section 4 we present and discuss the numerical results. Section 5 is devoted to the conclusion.

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2 The model

The isolated two-leg Hubbard ladder is described by the following dispersion relations linearized around the Fermi points [5,6]

$$\epsilon_k^m = \pm v_F^m \left(k \mp k_F^m\right), m = \mathrm{A, B} \tag{1}$$

where A (B) refers to the Antibonding (Bonding) band and k_F^m and v_F^m are respectively the Fermi point and the Fermi velocity in the band m. ϵ_k^m run over a range characterized by the bandwidth cutoff E_0. We assume that $v_F^m = v_F$ to avoid additional renormalization of the Fermi velocity [7].

The electrons interact through intraladder scattering processes characterized by the dimensionless constants $g_\mu^{(i)}, \mu = 0, f, t$ and $i = 0, 1$ within the g-ology model [6,8] where 0, f, t denote respectively intraband scattering, interband forward scattering and interband tunneling scattering and $g_\mu^{(1)}$ ($g_\mu^{(2)}$) refers to backward (forward) scattering (Fig. 3 of Ref. [6]).

Recent weak coupling RG study of isolated two-leg Hubbard ladder [5,6,9] revealed the existence of a strong coupling fixed point characterized by the opening of a spin gap. Such phase denoted by SGM (spin gap metal) in reference [6] is dominated by d-wave superconducting correlations.

The isolated ladders are weakly coupled by one and two particle hopping processes. The one particle hopping (two particle hopping) takes place when a single particle (a pair of particle) hops from one ladder to a neighboring one as it is illustrated in Figure 2 of reference [6].

The methodology used in this paper relies on the perturbative renormalization group approach (PRG), as discussed by Bourbonnais and Caron [10], which we generalize to the case of two cut-off's.

The application of PRG to the one dimensional and quasi-one dimensional electron gas has been extensively studied and discussed by Sólyom [8]. Following this work, a number of authors have used the same formalism with great success to discuss the properties of quasi-one dimensional conductors. For example the departure from commensurability of $2k_\mathrm{F}$ has been discussed by Seidel *et al.* [11] using PRG approach. Montambaux *et al.* [12] have discussed the Zeeman effect of a magnetic field with the same technique, while Japaridze *et al.* [13] found the same results by a bosonization method. Prigodin *et al.* [14] and Emery *et al.* [15] have discussed the decoupling of the zero-sound and Cooper channels caused by three dimensional coupling by studying the PRG differential equations. The PRG method has been modified when it is necessary to take care of a second energy scale involved in the problem, besides the bandwidth cut-off E_0 [12,16]. Here the second energy scale is introduced by the field, namely the magnetic energy $\omega_\mathrm{c} = evHd$ where d is the interladder distance and H is the field magnitude. The numerous examples which can be found in the literature of well known and well accepted works using this method seem to validate the use of the PRG approach to the problem of a two-leg ladder in a magnetic field.

Let us consider the effect of an external magnetic field $\mathbf{H} = (0, 0, H)$ transverse to the plane of the ladders. The interladder hopping hamiltonian is then given by

$$H_\perp = -t_\perp \sum_{m=\mathrm{A,B}} \sum_{<l,l'>} \int \mathrm{d}x e^{\mathrm{i}e\int_{x,ld}^{x,l'd} \mathbf{A}(\mathbf{s}).\mathbf{ds}} \psi_m^+(x,l)\psi_m(x,l') \tag{2}$$

where $\psi_m(x,l)$ ($\psi_m^+(x,l)$) is a particle annihilation (creation) operator, l is the ladder index, m refers to the band and x is the coordinate along the ladder direction.

We have chosen the following gauge

$$\mathbf{A} = (0, Hx, 0). \tag{3}$$

Consequently $H_\perp$ is written as

$$H_\perp = -t_\perp \sum_{m=\mathrm{A,B}} \sum_{<l,l'>} \int \mathrm{d}x e^{\mathrm{i}Gx(l'-l)} \psi_m^+(x,l)\psi_m(x,l') \tag{4}$$

where $G = eHd$. If we consider a mixed representation by taking the Fourier transform with respect to x, $H_\perp$ will be given by

$$H_\perp = -t_\perp \sum_{m=\mathrm{A,B}} \sum_{<l,l'>,k_\parallel} \psi_m^+(k_\parallel,l)\psi_m(k_\parallel + G(l'-l), l') \tag{5}$$

where $k_\parallel$ is the longitudinal momentum.

We see that in $H_\perp$, the magnetic vector $\mathbf{G} = (G, 0, 0)$ is coupled to the longitudinal momentum $k_\parallel$ in the phase of $\psi_m^+(k_\parallel, l')$ which express the orbital effect of the magnetic field.

Therefore it is only when a hop from one ladder to a neighboring one occurs that the translation in the phase of a particle operator takes place.

Since we are interested in the effect of a magnetic field on a system of coupled ladders, we will assume that the intraladder processes are not affected by the presence of the field. In fact to have an appreciable effect on the intraladder hopping processes, the magnetic energy ω_c should be of the same order of such processes. The amplitudes of the intraladder hopping terms are about $10t_\perp$ [17]. Using reasonable parameters for the Fermi velocity and the intraladder distances [18], we have estimated the magnitude of the magnetic field that will be able to overcome the intraladder hopping processes and we have found that it should be of the order of 500 T! However we will be interested in relatively weak magnetic fields which do not exceed 25 T. Under such magnetic fields the intraladder processes are really not affected.

3 Renormalization group equations

To derive the scaling equations for intraladder and interladder processes we have adopted the renormalization group formulation of Bourbonnais and Caron [10] based on Kadanoff-Wilson model. This approach was applied in the case of weakly coupled ladder in absence of magnetic field [6]. The details of calculation exist already in references [6,10].

Because we have assumed that intraladder processes are not affected by the magnetic field, the scaling equations of the two particle scattering are then unchanged and are given by equations (3.11-3.16) of reference [6].

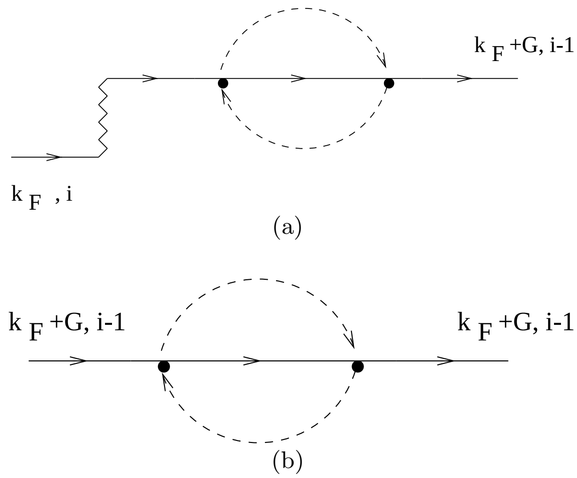


Fig. 1. Diagrammatic representations of the scaling equation of the one particle hopping process amplitude $t_\perp$ (denoted by a dark dotted line) (a) and of the second order correction to the self energy (b). The solid and broken lines represent the propagators for right-moving and left-moving electrons.

3.1 RG equations for one particle hopping process

The diagrammatic representation of the scaling equation of $t_\perp$ is shown in Figure 1a. It depends on the intraladder self energy correction represented in Figure 1b. Assuming that $\omega_c < E_0 \sim E_F$ (which is reasonable) we find that this correction diverges as $\mathrm{Log}[(\omega - \delta\omega_c)/E_0]$ where $\delta = \pm 1$ and $\omega_c = v_F G$. If the magnitude of the field tends to zero, we recover the logarithmic divergence in absence of a magnetic field that is $\mathrm{Log}(\omega/E_0)$.

This logarithmic term may be written as

$$\mathrm{Log}\left(\frac{\omega - \delta\omega_c}{E_0}\right) = \mathrm{Log}\left(\frac{\omega_c}{E_0}\right) + \mathrm{Log}\left(\frac{\omega}{\omega_c} - \delta\right). \quad (6)$$

We notice that the logarithmic divergence with respect to ω is left.

Since we are interested in accessible value of the field, $\omega_c/E_0 \ll 1$ and in the behavior of the system for $\omega \to 0$, we are left with a problem with two logarithmic singularities one in $\mathrm{Log}\,(\omega)$ (in the case of two particle vertices), which is well known, and the other in $\mathrm{Log}\,(\omega_c)$.

We should set up a double renormalization procedure with two cut-off parameters namely E_0 and ω_c. Therefore we choose ω_c/E_0 and ω/ω_c as the two independent scaling parameters, and this is the only choice.

This two cut-off renormalization group (RG) approach was applied in the case of the quasi-one dimensional conductors under magnetic field [12]. The two cut-offs were E_0 and the Zeeman energy.

It is worth to note that the logarithmic term in the two particle vertices is read as

$$\mathrm{Log}\left(\frac{\omega}{E_0}\right) = \mathrm{Log}\left(\frac{\omega_c}{E_0}\right) + \mathrm{Log}\left(\frac{\omega}{\omega_c}\right). \quad (7)$$

Therefore the scaling equations of the intraladder coupling constants $g_\mu^{(i)}$ are the same in the the two steps of RG.

We derive the RG equations related to the logarithmic problem in ω_c/E_0 keeping ω/ω_c constant while ω_c/E_0 scales from 1 to its physical value. We can call this renormalization step the filed renormalization as in reference [12].

This first step of RG gives rise to effective couplings which will be the scaling point of the second step of the renormalization where we vary ω/ω_c from 1 to 0 keeping ω_c/E_0 constant.

This step may be considered as a frequency renormalization.

The scaling equation of the dimensionless hopping amplitude $\tilde{t}_\perp(l) \equiv t_\perp(l)/E_0$ is given by [6,10]

$$\frac{\mathrm{dLog}\tilde{t}_\perp(l)}{\mathrm{d}l} = 1 - \frac{\mathrm{dLog}z_1}{\mathrm{d}l} \quad (8)$$

where l is the scaling parameter and is given by $l = \mathrm{Log}(E'/E_0)$ for the first step of the RG and by $l = \mathrm{Log}(E'/\omega_c)$ for the second step ($E' \equiv E(l)$ is the scaling energy). The second term in rhs of equation (8) is given by the correction to the self energy depicted in Figure 2 [10].

The scaling equation of $\tilde{t}_\perp(l)$ in the first RG step is then

$$\frac{\mathrm{dLog}\,\tilde{t}_\perp}{\mathrm{d}l} = 1 - (g_0^{(1)2} + g_0^{(2)2} - g_0^{(1)}g_0^{(2)} + g_f^{(1)2} + g_f^{(2)2} - g_f^{(1)}g_f^{(2)} + g_t^{(1)2} + g_t^{(2)2} - g_t^{(1)}g_t^{(2)}) \quad (9)$$

where

$$l = 0, ..., l_{\mathrm{max1}} \text{ and } l_{\mathrm{max1}} = -\mathrm{Log}[\max(T, \omega_c, T_{\mathrm{cross}})/E_0].$$

l_{max1} corresponds to the thermal fluctuations (T), one particle dimensionality crossover (T_{cross}) or magnetic energy (ω_c).

If $l_{\mathrm{max1}} = l_c \equiv \mathrm{Log}\,(E_0/\omega_c)$ then we will carry out the second step of the RG procedure where the scaling equation of $\tilde{t}_\perp(l)$ is reduced to

$$\frac{\mathrm{dLog}\tilde{t}_\perp(l)}{\mathrm{d}l} = 1 \quad (10)$$

where

$$l = 0, ..., l_{\mathrm{max2}} \text{ and } l_{\mathrm{max2}} = \mathrm{Log}[\max(T, T_{\mathrm{cross}})/E_0].$$

In fact as we have noted, in the second RG step the correction given by the self energy is proportional to $\mathrm{Log}(\omega/\omega_c - \delta)$ and so is not divergent. Thus the scaling of $\tilde{t}_\perp(l)$ is independent of such correction which yields to equation (10).

It is worth noting that in this step of RG the intraladder coupling constants $g_\mu^{(i)}$ rapidly reach their fixed values corresponding to the strong coupling phase (SGM). The convergence may be obtained for $l_{\mathrm{limit}} \approx 3$. Therefore if we choose the magnitude H of the field in such a way that $\tilde{t}_\perp(l_c) < 1$ in the first RG and $\tilde{t}_\perp(l_{\mathrm{limit}}) < 1$, so we could be able to stop the one hopping process and confine the electron motion within the ladder. The value of H yielding

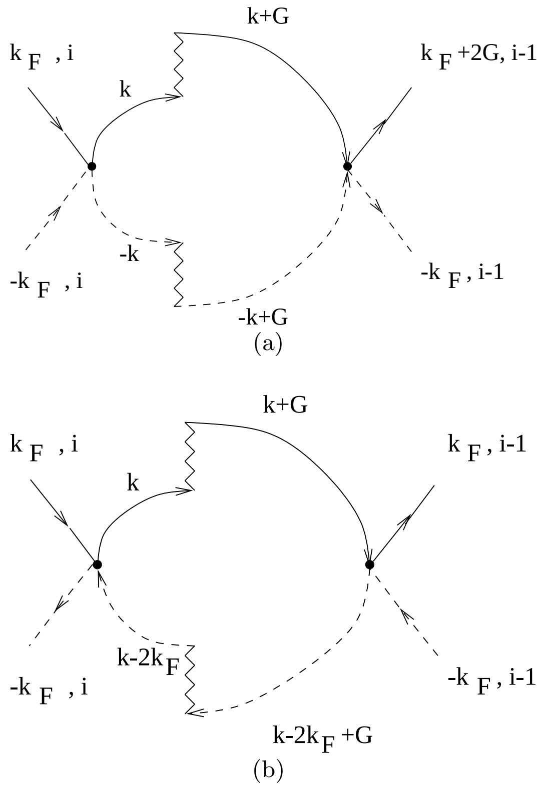


Fig. 2. Diagrammatic representation of the generators terms of the superconducting channel (SCd) (a) and the spin density wave channel (SDW) (b), (dark dotted lines denote $t_\perp$).

to confinement will increase with increasing $\tilde{t}_{\perp 0}$ which is the bare value of $\tilde{t}_\perp(l)$ in the first RG. Considering $\tilde{t}_{\perp 0}$ as an applied pressure [6], we can conclude that confinement will requires increasing magnitude of applied field as the pressure increases.

3.2 RG equations for two particle hopping process

The scaling equations for the interladder two particle hopping amplitude are depicted in reference [6] and we will study only the most dominant processes namely V_0^{SDW} and $V^{\mathrm{SC}d}$ corresponding to the intraladder spin density wave (SDW) and the d-wave superconducting channels (SCd).

For these two processes the scaling equations may be written as

$$\frac{\mathrm{d}V^{\mathrm{SC}d}(l)}{\mathrm{d}l} = f^{\mathrm{SC}d}(l) + 2g^{\mathrm{SC}d}V^{\mathrm{SC}d}(l) - \frac{1}{2}(V^{\mathrm{SC}d}(l))^2 \tag{11}$$

$$\frac{\mathrm{d}V_0^{\mathrm{SDW}}(l)}{\mathrm{d}l} = f^{\mathrm{SDW}}(l) + g^{\mathrm{SDW}}V_0^{\mathrm{SDW}}(l) - \frac{1}{2}(V_0^{\mathrm{SDW}}(l))^2 \tag{12}$$

where $g^{\mathrm{SC}d} = \frac{1}{2}(g_t^{(1)} + g_t^{(2)} - g_0^{(1)} - g_0^{(2)})$ and $g^{\mathrm{SDW}} = g_0^{(2)}$.

$f^{\mathrm{SC}d}$ and f^{SDW}, which are depicted in Figure 2, are the generating terms for SCd channel and the SDW channel respectively. The effect of the magnetic field in equations (11, 12) appears explicitly only in these generating terms.

From Figure 2, we see that for the SDW channel (electron-hole pair) the total energy is conserved after the second interaction whereas there is an energy gain of $2\omega_{\mathrm{c}}$ for the SCd channel. This gain will be provided by the thermal fluctuations. Hence for $T < \omega_{\mathrm{c}}$ the pair will be broken and the SCd transition never occurs [19].

We should note that the scaling equations are derived using the usual approximation $E(l) \gg T$ where $E(l)$ is the scaling energy. However the temperature dependence of the SCd generating term, $f^{\mathrm{SC}d}$, should be kept in order to take into account the thermal condition imposed by the external legs. $f^{\mathrm{SC}d}$ is then given by

$$\begin{aligned} f^{\mathrm{SC}d}(l) = &-C\tilde{t}_\perp^2(l)(g^{\mathrm{SC}d})^2 \cos(q_\perp)\,\frac{E(l)}{\omega_{\mathrm{c}}} \\ &\times\left[\tanh\left(\frac{E(l)}{4T} - \frac{\omega_{\mathrm{c}}}{2T}\right) - \tanh\left(\frac{E(l)}{4T} + \frac{\omega_{\mathrm{c}}}{2T}\right)\right] \\ &-C\tilde{t}_\perp^2(l)(g^{\mathrm{SC}d})^2 \cos(q_\perp)\,\frac{1}{1-\frac{\omega_{\mathrm{c}}}{E(l)}} \\ &\times\left[\tanh\left(\frac{E(l)}{4T}\right) + \tanh\left(\frac{E(l)}{4T} - \frac{\omega_{\mathrm{c}}}{2T}\right)\right] \\ &-C\tilde{t}_\perp^2(l)(g^{\mathrm{SC}d})^2 \cos(q_\perp)\,\frac{1}{1+\frac{\omega_{\mathrm{c}}}{E(l)}} \\ &\times\left[\tanh\left(\frac{E(l)}{4T}\right) + \tanh\left(\frac{E(l)}{4T} + \frac{\omega_{\mathrm{c}}}{2T}\right)\right] \end{aligned} \tag{13}$$

whereas the f^{SDW} is written as

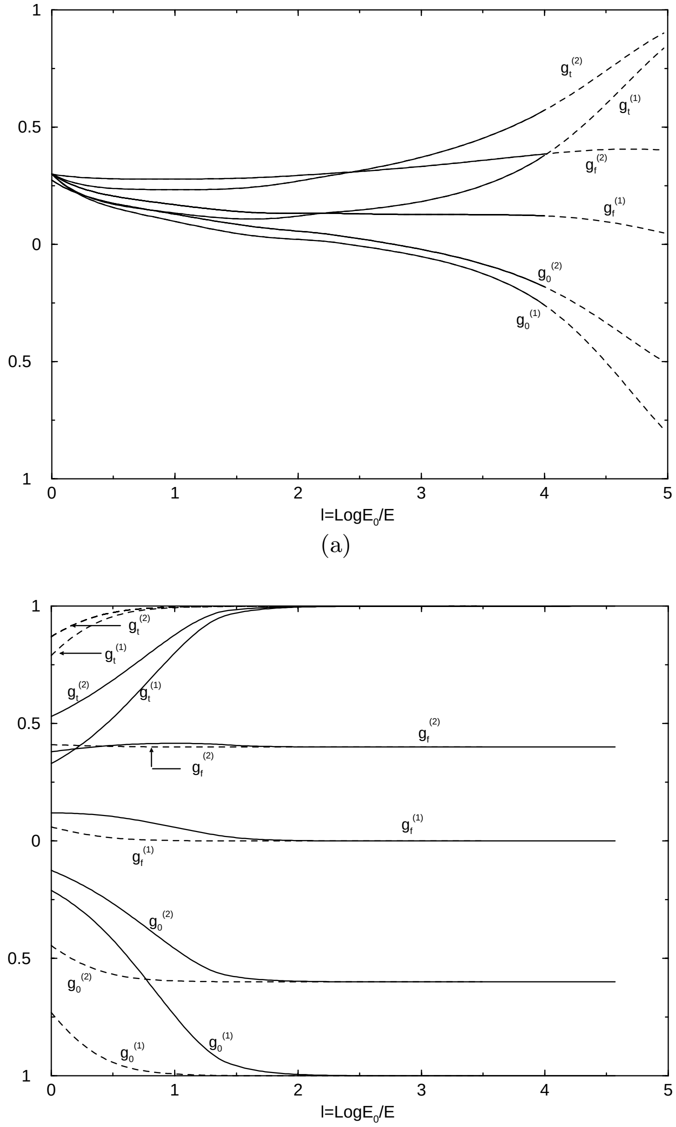
$$f^{\mathrm{SDW}}(l) = -\frac{1}{4}\tilde{t}_\perp^2(l)(g^{\mathrm{SDW}})^2\cos(q_\perp)\left[\frac{1}{1-\left(\frac{\omega_{\mathrm{c}}}{E(l)}\right)^2} - \frac{2}{1-\left(\frac{2\omega_{\mathrm{c}}}{E(l)}\right)^2}\right] \tag{14}$$

where $q_\perp = 0$ for the SCd channel and $q_\perp = \pi$ for the SDW channel.

The normalization factor C in equation (13) is given by

$$C = \left(4\tanh\left(\frac{E(l)}{4T}\right) + \frac{E(l)}{\omega_{\mathrm{c}}}\left[\tanh\left(\frac{E(l)}{4T} - \frac{\omega_{\mathrm{c}}}{2T}\right) - \tanh\left(\frac{E(l)}{4T} + \frac{\omega_{\mathrm{c}}}{2T}\right)\right]\right)^{-1}. \tag{15}$$

This factor does not emerge during the internal integration. It is just introduced to recover numerically the zero field case. Owing to the fact that $\omega_{\mathrm{c}} < E(l)$ for the first step of the RG procedure and $E(l) \gg T$, this term will reduce to the analytical normalization factor.



(b)

Fig. 3. Scaling flows of the coupling constants at the first step of the renormalization procedure (a) and at the second step (b) for $H = 1.6$ T (dashed lines) and $H = 7.3$ T (solid lines).

It is worth noting that $V^{\mathrm{SC}d}$ will scale to zero if $\omega_{\mathrm{c}} > T$. Hence, for the SCd channel, the first RG procedure should be stopped at $l_{\max} = -\mathrm{Log}[\max(T, T_{\mathrm{cross}})]$ and the second step is then useless.

From equation (14), we can note that the first RG should be carried out for the V_0^{SDW} up to $2\omega_{\mathrm{c}}$ due to the divergence of f^{SDW} at $2\omega_{\mathrm{c}}$.

Let us denote by f_1^{M} and f_2^{M} (M = SDW, SCd) the generating terms for the first and the second renormalization procedures. $f_1^{\mathrm{SC}d}$ will be obtained from equation (13) by setting $E(l) = E_0\mathrm{e}^{-l}$ and $\omega_{\mathrm{c}} = E_0\mathrm{e}^{-l_{\mathrm{c}}}$ if $\omega_{\mathrm{c}} < T$ and $f_1^{\mathrm{SC}d}$ will be equal to zero otherwise.

We should note that $f_2^{\mathrm{SC}d}$ becomes meaningless because we will stop the first RG for $V^{\mathrm{SC}d}$ at $T > \omega_{\mathrm{c}}$ due to thermal conditions discussed above.

f_i^{SDW}, $i = 1, 2$ are written as

$$f_1^{\mathrm{SDW}}(l) = \frac{1}{4}\tilde{t}_\perp^2(l)(g^{\mathrm{SDW}})^2 \left[\frac{1}{1-\mathrm{e}^{2(l-l_{\mathrm{c}})}} - \frac{2}{1-4\mathrm{e}^{2(l-l_{\mathrm{c}})}}\right] \quad (16)$$

and

$$f_2^{\mathrm{SDW}}(l) = \frac{1}{4}\tilde{t}_\perp^2(l)(g^{\mathrm{SDW}})^2 \left[\frac{1}{1-\frac{1}{4}\mathrm{e}^{2l}} - \frac{2}{1-\mathrm{e}^{2l}}\right]\cdot \quad (17)$$

The numerical integration of $V^{\mathrm{SC}d}$ and V^{SDW} reveals that V^{SDW} is much smaller than $V^{\mathrm{SC}d}$ for the choice of coupling constant given below. Henceforth we will be interested only in $V^{\mathrm{SC}d}$ as the most dominant two particle process.

We should note that if we develop f^{SDW} (Eq. (14)) with respect to $\omega_{\mathrm{c}}/E(l)$ we will recover the result of reference [20] in the case of organic conductors under weak magnetic field.

4 Results and discussion

We have carried out the numerical integration of equations (9, 13) and equations ((3.11)-(3.16)) of reference [6] for the first step of the RG. We have used the the following bare values

$$g_\mu^{(i)}(0) = 0.3,\ V^{\mathrm{SC}d}(0) = 0,\ \tilde{t}_\perp(0) = \tilde{t}_{\perp 0} \quad (18)$$

$\tilde{t}_{\perp 0}$ is regarded as an applied pressure [6].

In order to estimate the magnitude of the magnetic field, we have taken $v_{\mathrm{F}} = 10^6\mathrm{ms}^{-1}$ and $d = 3$ Å for the Fermi velocity and the interladder distance respectively. These values are reasonable for the superconducting ladder materials such as $\mathrm{Sr}_{14-x}\mathrm{Ca}_x\mathrm{Cu}_{24}\mathrm{O}_{41}$ [17].

In Figure 3 we have depicted the scaling flows of $g_\mu^{(i)}$ within the first renormalization step (a) and the second one(b) for $H = 1.6$ T and $H = 7.3$ T.

Carrying out the first renormalization step procedure we may meet three situations:

(a) at $E(l) > T$, $\tilde{t}_\perp(l)$ reaches unity before $V^{\mathrm{SC}d}$ diverges, then we should stop the renormalization procedure because the system will crosses over to the two dimensional (2D) Fermi liquid phase. It is the case where $\omega_{\mathrm{c}} < T < T_{\mathrm{cross}}$, T_{cross} being the one particle two dimensional crossover temperature (denoted by T_{x_1} in Ref. [10]).

(b) at $E(l) > T$, $V^{\mathrm{SC}d}$ diverges before $\tilde{t}_\perp(l)$ reaches unity. In this case the system undergoes a phase transition to the SCd phase and we will stop the renormalization procedure too. This case is met when $T_{\mathrm{c}} > T > \omega_{\mathrm{c}}$, T_{c} is the two particle dimensionality crossover temperature (denoted by T_{x_2} in Ref. [10]).

(c) at $E(l_{\max}) \approx T$, $\tilde{t}_\perp(l)$ is smaller than unity and $V^{\mathrm{SC}d}$ is not divergent. In this case the one and two particle processes are stopped and the system is left in the isolated ladder phase (SGM).

It should be noted that for the SDW channel, if at $l_{\max} \approx -\mathrm{Log}\,(2\omega_c/E_0)$ $\tilde{t}_\perp(l)$ is smaller than unity and V^{SDW} is not divergent then we have to carry out the second step of renormalization which starts with the renormalized value of $g_\mu^{(i)}(l)$, $\tilde{t}_\perp(l)$ and $V^{\mathrm{SDW}}(l)$ at $l = l_{\max}$, namely

$$\left[g_\mu^{(i)}(0)\right]_2 = \left[g_\mu^{(i)}(l_{\max})\right]_2, \ \left[\tilde{t}_\perp(0)\right]_2 = \left[\tilde{t}_\perp(l_{\max})\right]_1, \\ \left[V^{\mathrm{SDW}}(0)\right]_2 = \left[V^{\mathrm{SDW}}(l_{\max})\right]_1 \tag{19}$$

where the labels 1 and 2 denote respectively first and second renormalization step procedure.

In order to get the phase diagram (T-$\tilde{t}_{\perp 0}$) (which may be regarded as temperature-pressure diagram) we have to solve the scaling equations for different values of $\tilde{t}_{\perp 0}$ and for a fixed value H of the magnetic field.

Figure 4a-e show phase diagrams obtained for $H =$ 0 T(a), 1.6 T(b), 2.7 T(c), 3.7 T(d) and 12 T(e). $\tilde{T}_c$ and $\tilde{T}_{\mathrm{cross}}$ (denoted by T'_c and T'_{cross} in the figures) are given as in reference [6] by

$$\tilde{T}_{\mathrm{cross}} = \frac{T_{\mathrm{cross}}}{E_0} = \mathrm{e}^{-l_{\mathrm{cross}}}, \ \tilde{T}_c = \frac{T_c}{E_0} = \mathrm{e}^{-l_{\mathrm{SC}d}} \tag{20}$$

where $\tilde{t}_\perp(l_{\mathrm{cross}}) = 1$ and $V^{\mathrm{SC}d}(l_{\mathrm{SC}d}) = -\infty$.

As we can note, as the field increases the SGM phase is established at the expense of the SCd phase which shrinks and its starting point gets shifted to higher pressure. For a critical value of the magnetic field $H_{\mathrm{SC}d}$ the SCd phase is completely destroyed.

Concerning the 2D phase, we see that for $H < H_{\mathrm{SC}d}$ (corresponding to $\omega_c < T_{\mathrm{cross}}$), this phase is unaffected. However for $H > H_{\mathrm{SC}d}$ and for increasing magnetic field the 2D phase gets smaller and is replaced by the SGM phase.

It is interesting to study the evolution of the superconducting temperature T_c with the magnetic field. Figure 5 shows the dependence of T_c on the the magnitude H of the field for different values of $\tilde{t}_{\perp 0}$ (pressure). We note that $T_c(H)$ decreases with H and falls to zero at a critical value H_c which increases with pressure as it is shown in Figure 6.

The decrease of T_c with increasing H reflects the confinement of particle motion within the ladders. The fall of $T_c(H)$ to zero may be explained as follows. We have seen in Section 3.1 that for the SCd channel the pair is broken at $T < \omega_c$. Therefore if the SCd transition occurs, $V^{\mathrm{SC}d}$ should diverge at $T > \omega_c$. With increasing H, the RG procedure should be stopped at earlier stage. For a critical value H_c of the field $V^{\mathrm{SC}d}$ does not diverge at $l < l_{\max} = \mathrm{Log}\left(\frac{E_0}{T}\right)$ where $T > \omega_c = ev_F d H_c$ and then the SCd transition does not occur. Thus $T_c(H)$ falls to zero.

The growth of H_c with $\tilde{t}_{\perp 0}$ (Fig. 6) expresses the competition between the magnetic field and pressure. The unidimensionalization of particle motion requires increasing magnetic field as pressure increases.

In Figure 7 we have represented T_c with respect to $\tilde{t}_{\perp 0}$ (pressure) for different values of H. We note that for a given value of the field, T_c increases with pressure as it is expected. This behavior is met in the case of zero magnetic field [2].

To summarize these results we have depicted the $(T, \tilde{t}_{\perp 0}, H)$ phase diagram in Figure 8.

We can conclude that the magnetic field reduces the interladder one and two particle hopping processes. The particle motion is then unidimensionalized within the ladder which explain the appearance of the SGM phase at low temperature. Therefore as the field increases the coupling between ladders is decreased and the system, which may be in the SCd phase or the 2D phase at zero magnetic field, will scale to the isolated ladder phase (SGM phase). The effective dimensionality of the system is then reduced by applying a magnetic field.

It is worth noting that for the range of $\tilde{t}_{\perp 0}$ where the 2D phase appear, a field, for which $\omega_c > \tilde{T}_{\mathrm{cross}}$, will be able to stop the crossover to the 2D phase. For the values of $\tilde{t}_{\perp 0}$ depicted in the phase diagrams, we have found that a field of about 20 T will destroy completely the 2D phase and the SCd phase found at zero magnetic field and the corresponding phase diagram is reduced to the SGM phase for any value of $\tilde{t}_{\perp 0}$. Such field is accessible, which give us an opportunity to have an experimental test to our model.

We should remark that it will be interesting to study the $T < T_{\mathrm{cross}}$ range by taking into account the two dimensional effects in the RG equations [21]. The bare coupling constants will be the renormalized constants at T_{cross}. In this two dimensional phase the nesting properties of the Fermi surface should be taken into account when calculating the electron-electron and electron-hole bubbles for the RG equations. These nesting properties may reveal some interesting features such as a competition between the SCd channel and the SDW channel which may be restored due to good nesting conditions. Nesting deviations may also be introduced by including second neighbors ladders hopping processes $t_{\perp 2}$ that may be bypassed by the magnetic field which is known to restore good nesting conditions. This procedure was already applied in the case of quasi-one organic conductors under magnetic field [20] and has permitted to recover the FISDW phases [3,4]. In our case, however, specific effects arise because of the two energy bands present in the ladder compounds. However, since such study gets over the scope of this paper, these effects will not be discussed here. A forthcoming paper will be devoted to them.

5 Conclusion

We have studied the orbital effect of the magnetic filed on the two-leg Hubbard ladder weakly coupled by one particle hopping process. We have assumed that the intraladder processes are not affected by the field.

Using a perturbative renormalization group approach (PRG) with two cut-off parameters, the bandwidth E_0 and the magnetic energy ω_c, we have studied the relative

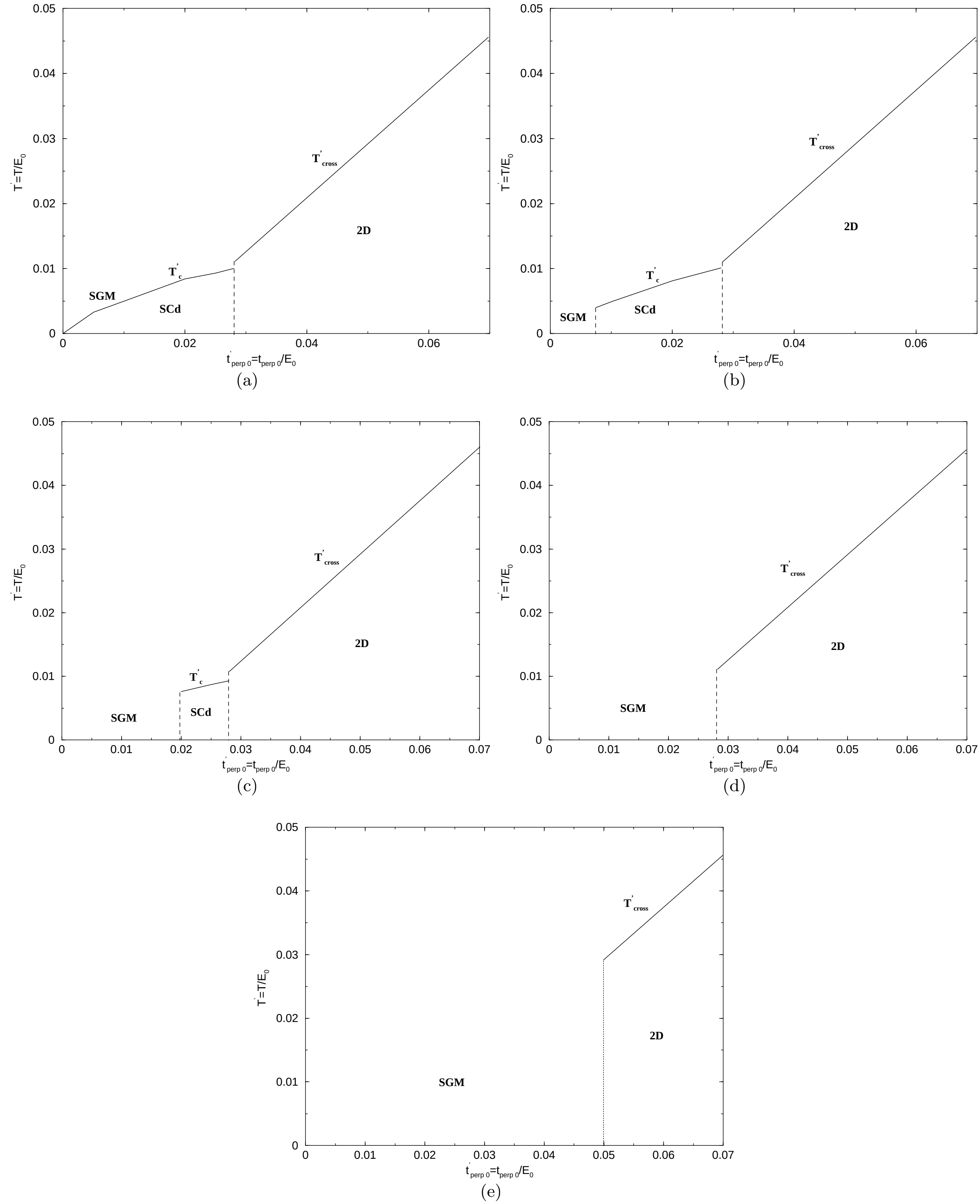


Fig. 4. Phase diagram of weakly coupled Hubbard ladder under a magnetic field $H = 0$ T (a), 1.6 T (b), 2.7 T (c), 3.7 T (d) and 12 T (e).

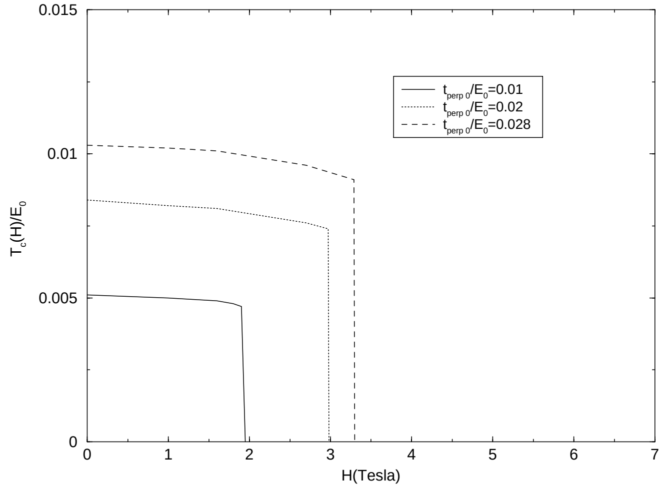


Fig. 5. Superconducting temperature T_c *versus* the applied magnetic field for different values of $\tilde{t}_{\perp 0}$ ($t_{\text{perp}0}$ is denoted by $t_{\text{perp}0}$ in the figure).

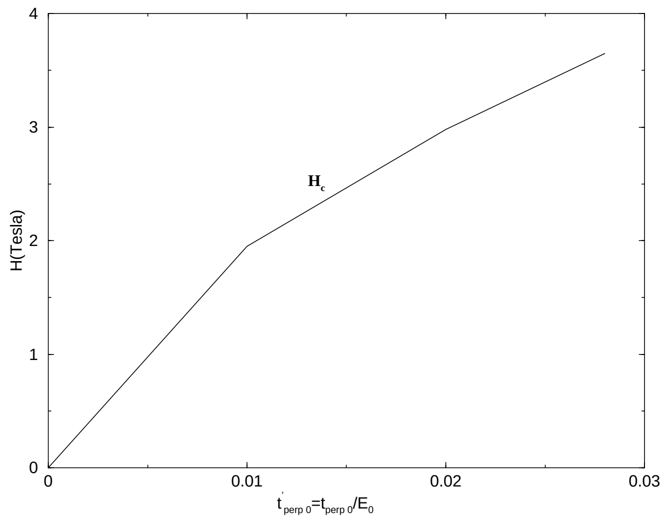


Fig. 6. Critical magnetic field *versus* $\tilde{t}_{\perp 0}$.

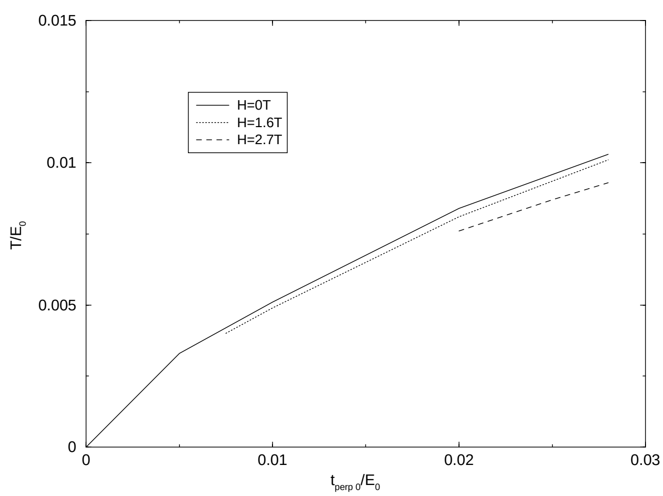


Fig. 7. Superconducting temperature $\tilde{T}_c$ *versus* $\tilde{t}_{\perp 0}$ (pressure) for different values of the magnetic field.

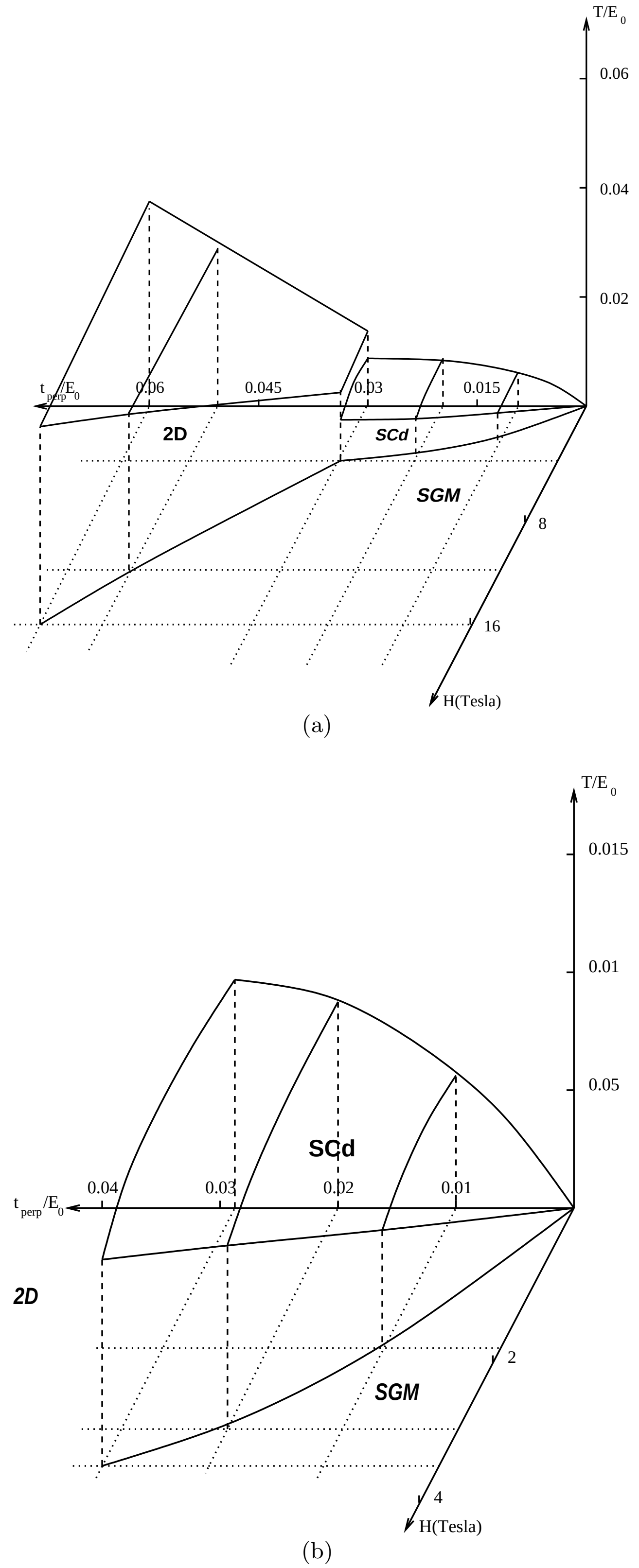


Fig. 8. $(\tilde{T}, \tilde{t}_{\perp 0}, H)$ phase diagram for $\tilde{t}_{\perp 0} < 0.06$ (a) and for $\tilde{t}_{\perp 0} < 0.03$ (b).

stability of the d-wave superconducting phase (SCd) and the two dimensional (2D) Fermi liquid phase.

We have shown that the SCd phase shrinks with increasing magnetic filed and gets shifted to higher pressure. The critical temperature $T_c(H)$, at which the SCd transition occurs, decreases with increasing field up to a critical value H_c where it falls to zero and so the SCd transition will not occur. We have found that for a given magnetic field T_c increases with pressure as it is the case of zero magnetic filed.

Concerning the 2D phase, we have shown that for a field such as $\omega_c > T_{\text{cross}}$, (where T_{cross} is the crossover temperature), we are able to stop the crossover and leave the system in the isolated ladder phase (SGM). We think that a reasonable value of the field of about 20 T will be sufficient to stop the one and two particle hopping processes and drive the system to the isolated ladder phase. In this case the electron motion will be confined within the ladder.

S. Haddad is grateful to C. Bourbonnais, S. Kaddour and K. Frikach for stimulating discussions and would like to thank le Centre de Recherche en Physique du Solide à l'université de Sherbrooke for hospitality and financial support.

References

1. E. Dagotto, T.M. Rice, Science **271**, 619 (1995).
2. M. Uehara, T. Nagato, J. Akimitsu, H. Takahashi, N. Môri, K. Kinoshita, J. Phys. Soc. Jap. **65**, 2764 (1996).
3. L.P. Gor'kov, A.G. Lebed, J. Phys. Lett. France **45**, L433 (1984).
4. M. Héritier, G. Montambaux, P. Lederer, J. Phys. Lett. France **45**, L943 (1984).
5. M. Fabrizio, Phys. Rev. B **48**, 15838 (1993).
6. J. Kishine, K. Yonemitsu, J. Phys. Soc. Jap. **67**, 1714 (1998).
7. K. Penc, J. Sólyom, Phys. Rev. B **41**, 704 (1990).
8. J. Sólyom, Adv. Phys. **28**, 201 (1979).
9. H.H. Lin, L. Balents, M.P.A. Fisher, Phys. Rev. B **56**, 6569 (1997).
10. C. Bourbonnais, L.G. Caron, Int. J. Mod. Phys. B **5**, 1033 (1991).
11. C. Seidel, V.M. Prigodin, J. Low Temp. Phys. **53**, 419 (1983).
12. G. Montambaux, M. Héritier, P. Lederer, J. Phys. Lett. France **45**, L533 (1984).
13. G.I. Japaridze, A.A. Nersesyan, J. Low Temp. Phys. **37**, 95 (1979).
14. V.N. Prigodin, Y.A. Firsov, J.E.T.P. Lett. **25**, 79 (1977).
15. V.J. Emery, R. Bruinsma, S. Barisić, Phys. Rev. Lett. **48**, 1039 (1982).
16. G.S. Grest, E. Abrahams, S.T. Chui, P.A. Lee, A. Zawadowiski, Phys. Rev. B **14**, 1225 (1976).
17. T.F.A. Muller, V. Anisimov, T.M. Rice, I. Dasgupta, T. Saha-Dasgupta, `cond-mat/9801280`; B. Normand, D.F. Agterberg, T.M. Rice, `cond-mat/9812211`.
18. E. Orignac, T. Giamarchi, `cond-mat/ 9704064`; Y. Mizuno, T. Tohyama, S. Maekawa, J. Phys. Soc. Jap. **66**, 937 (1997).
19. L. Hubert, Thesis, Université de Sherbrooke, 1997 (unpublished).
20. L. Hubert, C. Bourbonnais, Synth. Met. **55-57**, 4231 (1993).
21. C. Bourbonnais, Strongly interacting Fermions and HTS Superconductors, *Proc. of École de Physique Théorique, les Houches, Session LVI, 1991*, edited by B. Douçot, J. Zinn-Justin (Elsevier Science, 1995), p. 307.